The Neutral Kaons

Originally, the existence of the anti neutral kaon was not known.

So when they looked at the kaons they saw the set with strangeness...

\[ S(K^+) = +1 \quad S(K^-) = -1 \]

\[ S(K^0) = +1 \quad ? \]

Gell-Mann suggested that the \( K^0 \) should have an antiparticle with strangeness \( S = -1 \).

This anti kaon was found in the reaction:

\[ p + \pi^+ \rightarrow p + K^+ + \bar{K}^0 \quad \text{This reaction has } \Delta S = 0 \]

The observed neutral kaon can not be a \( K^0 \) since that would imply \( \Delta S = 2 \) which is assumed to be not allowed in Gell-Mann’s strangeness scheme – even for the weak interactions. (\( \Delta S = \pm 1 \) for the weak interactions.)
The Neutral Kaons

The two neutral kaons have a very unique property. They differ only in strangeness $S$ (and $T_3$) and nothing else. This is a fairly unique situation.

Both the $K^0$ and the $\bar{K}^0$ can decay to two pions via the weak interaction.

\[ K^0 \rightarrow \pi^+ \pi^- \quad \text{and} \quad \bar{K}^0 \rightarrow \pi^+ \pi^- \]

Both have $|\Delta S| = 1$

Therefore there can be a process that connects the two kaons, such as:

We can think of these as virtual pions.
The Neutral Kaons

Therefore it is possible to have the neutral kaons “oscillate” between being the particle and the antiparticle.

i.e. \( K^0 \rightarrow \bar{K}^0 \rightarrow K^0 \rightarrow \cdots \) via the weak interaction.

The particles are distinct at the time of production. This is because they are produced by the strong interaction in which strangeness is conserved.

i.e. \( p + \pi^- \rightarrow \Lambda + K^0 \)
\( p + \pi^+ \rightarrow p + K^+ + \bar{K}^0 \)

If the weak interaction could be turned off, then the \( K^0 \) and \( \bar{K}^0 \) would remain distinct.

So, after production, we cannot distinguish between the \( K^0 \) and \( \bar{K}^0 \).
The Neutral Kaons

When Gell-Mann proposed the strangeness scheme, Fermi said “I won’t believe in your scheme until you have a way of distinguishing the $K^0$ from the $\bar{K}^0$.”

Because, when we observe the decay of the neutral kaon, we cannot tell if it is a $K^0$ or a $\bar{K}^0$, maybe it is really some other particle state, which is a linear superposition of the $K^0$ and $\bar{K}^0$.

Equivalently we can consider the neutral kaons as being linear superpositions of other states.

This may not be such a strange idea if we think of the kaons as not fundamental particles, but just different states of a system.

What kind of particle states would they be?
We have seen that parity is not conserved in the weak interactions.

e.g. in the reaction $\pi^+ \rightarrow \mu^+ + \nu_\mu$

the muon always comes out with “left handed” helicity.

i.e.

Left handed muon

Right handed Helicity $\iff$ Projection of spin on momentum direction is positive i.e. in the same direction as the momentum.

Left handed Helicity $\iff$ Projection of spin on momentum direction is negative i.e. in the opposite direction as the momentum.
CP Invariance

It is through experiments such as the decay of the pion
\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
that we know that all neutrinos have left handed helicity and all antineutrinos have right handed helicity.

i.e.

Indeed, if the neutrino is massless, then a right handed neutrino can never be observed.
Since no observer can be moving faster than the neutrino to make it appear that the spin is in the same direction as the momentum. Then the helicity would be Lorentz invariant.
CP Invariance

Charge conjugation is also not conserved in the weak interactions.

For example, the charge conjugated version of the $\pi^+$ decay is

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

But anti neutrinos are right handed so in fact the muon is actually observed to be right handed in this reaction.
Therefore Parity and Charge conjugation are separately not conserved in the weak interaction.

But if we define the “mirror image” operation to be one that swaps particles with antiparticles as well as inverting the coordinate system, then this operation would be conserved in the weak interaction, as well as the strong and EM interaction.

This operation would be equivalent to applying the parity operation (as we have defined it) and the charge conjugation operation.

i.e. applying the operator $\hat{C}\hat{P}$

The conservation of CP would be consistent with the CPT theorem, assuming that time reversal is conserved.
CP Invariance

e.g.

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

left handed neutrino

left handed muon

Applying \( \hat{C} \Rightarrow \)

\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \]

left handed antineutrino

left handed antimuon

Applying \( \hat{P} \Rightarrow \)

\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \]

right handed antineutrino

right handed antimuon

Not observed.

Observed.
CP Invariance

For all neutrinos:

\[ \nu_L \xrightarrow{\hat{P}} \nu_R \]

Only these are observed in nature.

Both parity and charge conjugation are maximally violated. i.e. It is not that there is a preference for left-handed neutrinos over right-handed neutrinos – right handed neutrinos are never observed. Similarly left-handed anti-neutrinos are never observed.
The Neutral Kaons

For the neutral kaons:  
\[ \hat{CP} \qty| K^0 \rangle = \hat{C}(-1) \qty| K^0 \rangle = -\qty| \bar{K}^0 \rangle \]
\[ \hat{CP} \qty| \bar{K}^0 \rangle = \hat{C}(-1) \qty| \bar{K}^0 \rangle = -\qty| K^0 \rangle \]

Thus the neutral kaons are not eigenstates of \( \hat{CP} \)

i.e. \( \hat{CP} \qty| K^0 \rangle \neq \pm \qty| K^0 \rangle \)

Perhaps we should be considering the \( K^0 \) and \( \bar{K}^0 \) as linear combinations of states that are eigenstates of \( \hat{CP} \).

Defining:
\[ \qty| K_1 \rangle = \frac{1}{\sqrt{2}} \qty[ \qty| K^0 \rangle - \qty| \bar{K}^0 \rangle ] \]
\[ \qty| K_2 \rangle = \frac{1}{\sqrt{2}} \qty[ \qty| K^0 \rangle + \qty| \bar{K}^0 \rangle ] \]

Then
\[ \hat{CP} \qty| K_1 \rangle = \qty| K_1 \rangle \]
\[ \hat{CP} \qty| K_2 \rangle = -\qty| K_2 \rangle \]

i.e. \( K_1 \) and \( K_2 \) are eigenstates of both \( \hat{CP} \) and of the Hamiltonian of the system.

i.e. \( CP \) is conserved.
The Neutral Kaons

The relationship between $K_1$ and $K_2$, and $K^0$ and $\bar{K}^0$ is analogous to regarding linearly polarized light as a superposition of left and right polarized light of equal amplitude and definite phase.

So far we have considered $K^0$ and $\bar{K}^0$ as the "real particles", and $K_1$ and $K_2$ as the "derived particles". Do $K_1$ and $K_2$ actually exist as real particles? i.e. particles whose signature we can observe.

Consider their decay of the neutral kaon:

We know that $K^0 \rightarrow \pi^+ + \pi^-$

We first consider the angular momentum.

$$\vec{J}_i = 0 \quad \vec{J}_f = \vec{L}_{rel} + \vec{s}_\pi + \vec{s}_\pi \Rightarrow l_{rel} = 0$$

since $\vec{s}_\pi = 0$

$l_{rel}$ is the relative angular momentum q.n. in the final state.
The Neutral Kaons

\[ K^0 \rightarrow \pi^+ + \pi^- \]

For the final state:
\[ \hat{C}\hat{P}\ket{\pi^+\pi^-} = \hat{C}(-1)(-1)\ket{\pi^+\pi^-} \quad \text{since} \ l_{rel} = 0 \]
\[ \hat{C}\hat{P}\ket{\pi^+\pi^-} = \hat{C}\ket{\pi^+\pi^-} = \ket{\pi^+\pi^-} \quad \text{since} \ \hat{C}\ket{\pi^+} = \ket{\pi^-} \]
\[ \quad \text{and} \ \hat{C}\ket{\pi^-} = \ket{\pi^+} \]

i.e. \( CP = +1 \) in the final state.

Therefore since
\[ \hat{C}\hat{P}\ket{K_1} = \ket{K_1} \]
\[ \hat{C}\hat{P}\ket{K_2} = -\ket{K_2} \]

and we assume \( CP \) is conserved in this weak decay, the two pion final state can only come from the decay of a \( K_1 \).

This applies also for the decay \( K_1 \rightarrow \pi^0 + \pi^0 \)

So \( K_1 \rightarrow 2\pi \)
The Neutral Kaons

For the decay:  \( K^0 \rightarrow \pi^+ + \pi^- + \pi^0 \)

For the final state:
\[
\hat{CP} |\pi^+ \pi^- \pi^0 \rangle = \hat{C}(-1)(-1)(-1) |\pi^+ \pi^- \pi^0 \rangle \quad \text{since } l_{rel} = 0
\]
\[
\hat{CP} |\pi^+ \pi^- \pi^0 \rangle = -\hat{C} |\pi^+ \pi^- \pi^0 \rangle = -|\pi^+ \pi^- \pi^0 \rangle \quad \text{since } \hat{C} |\pi^\pm \rangle = |\pi^\pm \rangle
\]

i.e. \( CP = -1 \) in the final state.

Therefore since  \( \hat{CP} |K_1 \rangle = |K_1 \rangle \)
\[
\hat{CP} |K_2 \rangle = -|K_2 \rangle
\]

and we assume CP is conserved in this weak decay, the three pion final state can only come from the decay of a \( K_2 \).

This applies also for the decay \( K_2 \rightarrow \pi^0 + \pi^0 + \pi^0 \)

So \( K_2 \rightarrow 3\pi \)
The Neutral Kaons

So, there is a way to tell if the decay is from a $K_1$ or a $K_2$ but we cannot distinguish between a $K^0$ and $\bar{K}^0$.

It is expected that the $2\pi$ decay will be faster than the $3\pi$ decay because the energy released is greater.

\[
\begin{align*}
K_1 &\rightarrow 2\pi & T_{\frac{1}{2}} &= \text{short} \\
K_2 &\rightarrow 3\pi & T_{\frac{1}{2}} &= \text{long}
\end{align*}
\]

Some books label: $K_1 = K_S$  
$K_2 = K_L$

But we will see that these are slightly different later.

When this was predicted in 1956, only one half life for the neutral kaon was known ($\sim 10^{-10}$ s).

A search was made and indeed a second half life ($\sim 5 \times 10^{-8}$ s) was found.

The neutral kaon has two half lives!

(Because it is really a mixture of two particle states.)
The Neutral Kaons

Inverting the previous equations we get:

\[ \ket{K^0} = \frac{1}{\sqrt{2}} [\ket{K_1} + \ket{K_2}] \]

These decay to \(2\pi\) with \(T_{1/2} \sim 10^{-10}\) s

\[ \ket{K_1} = \frac{1}{\sqrt{2}} [\ket{K^0} - \ket{\bar{K}^0}] \]
\[ \ket{K_2} = \frac{1}{\sqrt{2}} [\ket{K^0} + \ket{\bar{K}^0}] \]

These decay to \(3\pi\) with \(T_{1/2} \sim 5 \times 10^{-8}\) s

Similarly:

\[ \ket{\bar{K}^0} = \frac{1}{\sqrt{2}} [\ket{K_1} - \ket{K_2}] \]

Since \(K_1\) and \(K_2\) have separate half lives and decay modes we should perhaps consider them to be the “real” particles. Furthermore the \(K_1\) and \(K_2\) have slightly different masses. The \(K^0\) and \(\bar{K}^0\) are mixtures of these with a well defined phase relationship.

But \(K^0\) and \(\bar{K}^0\) are antiparticles of each other; the \(K_1\) and \(K_2\) are not.
The Neutral Kaons

Let us suppose we produce a beam of $K^0$ particles, say by

$$\pi^- + p \rightarrow \Lambda + K^0$$

After sufficient flight time the $K_1$'s have all decayed away and we are left with only $K_2$. This is a mixture of $K^0$ and $\bar{K}^0$.

Therefore, just after production, at position A, $\Lambda$ particles cannot be produced by allowing the beam to hit a proton target:

$$K^0 + p \rightarrow \pi^+ + \Lambda \quad \Delta S = -2$$

But after drifting for a while, at position B, $\Lambda$ particles can be produced by:

$$\bar{K}^0 + p \rightarrow \pi^+ + \Lambda \quad \Delta S = 0$$
The Neutral Kaons

We can use an experimental arrangement like this to measure the number of $K^0$ at a time $t$ after production of $K^0$ particles.

Since the $K_1$ decays with decay constant $\lambda_1 = \frac{\ln 2}{T_{1/2,1}}$, we can write

$$|\psi(K_1, t)|^2 = |\psi_0 e^{-iE_t/\hbar}|^2 e^{-\lambda_1 t}$$

Number of $K_1$ at $t$ = Number of $K_1$ if they did not decay
$\psi_0$ = time independent part of $K_1$ wave function

So (switching notation) the time-dependent wave function may be written

$$|K_1, t\rangle = |K_1\rangle e^{-iE_t/\hbar} e^{-\lambda_1 t/2}$$

time independent part of $K_1$ wave function
The Neutral Kaons

$E_1$ is the energy of the $K_1$ in its rest frame, therefore $E_1 = m_1 c^2$

Therefore $|K_1, t\rangle = |K_1\rangle e^{-im_1 c^2 t/\hbar} e^{-\lambda_1 t/2}$

Similarly $|K_2, t\rangle = |K_2\rangle e^{-im_2 c^2 t/\hbar} e^{-\lambda_2 t/2}$

Now since: $|K^0\rangle = \frac{1}{\sqrt{2}} [\langle K_1\rangle + \langle K_2\rangle]$

$|K^0, t\rangle = \frac{1}{\sqrt{2}} \left\{ |K_1\rangle e^{-im_1 c^2 t/\hbar} e^{-\lambda_1 t/2} + |K_2\rangle e^{-im_2 c^2 t/\hbar} e^{-\lambda_2 t/2} \right\}$

But we also know that $|K_1\rangle = \frac{1}{\sqrt{2}} [\langle K^0\rangle - \langle \bar{K}^0\rangle]$

$|K_2\rangle = \frac{1}{\sqrt{2}} [\langle K^0\rangle + \langle \bar{K}^0\rangle]$

$|K^0, t\rangle = \frac{1}{2} \left\{ \left( |K^0\rangle - |\bar{K}^0\rangle \right) e^{-im_1 c^2 t/\hbar} e^{-\lambda_1 t/2} + \left( |K^0\rangle + |\bar{K}^0\rangle \right) e^{-im_2 c^2 t/\hbar} e^{-\lambda_2 t/2} \right\}$

$= \frac{1}{2} |K^0\rangle \left( e^{-im_1 c^2 t/\hbar} e^{-\lambda_1 t/2} + e^{-im_2 c^2 t/\hbar} e^{-\lambda_2 t/2} \right) + \frac{1}{2} |\bar{K}^0\rangle \left( -e^{-im_1 c^2 t/\hbar} e^{-\lambda_1 t/2} + e^{-im_2 c^2 t/\hbar} e^{-\lambda_2 t/2} \right)$
The Neutral Kaons

We can write

\[ |K^0, t\rangle = a(t) |K^0\rangle + b(t) |\bar{K}^0\rangle \]

\(a(t)\) is the probability amplitude for finding a \(K^0\) at time \(t\).

\(b(t)\) is the probability amplitude for finding a \(\bar{K}^0\) at time \(t\).

\[ a(t) = \frac{1}{2} \left( e^{-im_1c^2t/\hbar} e^{-\lambda_1t/2} + e^{-im_2c^2t/\hbar} e^{-\lambda_2t/2} \right) \]

\[ b(t) = \frac{1}{2} \left( -e^{-im_1c^2t/\hbar} e^{-\lambda_1t/2} + e^{-im_2c^2t/\hbar} e^{-\lambda_2t/2} \right) \]

Note: at \(t = 0\), \(a(0) = 1\) and \(b(0) = 0\) as it should.

\[ b(t) = \frac{1}{2} e^{-im_2c^2t/\hbar} \left(- e^{-i\Delta mc^2t/\hbar} e^{-\lambda_1t/2} + e^{-\lambda_2t/2} \right) \quad \text{with} \quad \Delta m = m_1 - m_2 \]

Therefore the probability of finding a \(\bar{K}^0\) at time \(t\) is

\[ P(\bar{K}^0) = |b(t)|^2 = \frac{1}{4} \left( e^{-\lambda_1t} + e^{-\lambda_2t} - e^{-(\lambda_1+\lambda_2)t/2} \left( e^{-i\Delta mc^2t/\hbar} + e^{+i\Delta mc^2t/\hbar} \right) \right) \]

\[ = \frac{1}{4} \left( e^{-\lambda_1t} + e^{-\lambda_2t} - 2e^{-(\lambda_1+\lambda_2)t/2} \cos(\Delta mc^2t/\hbar) \right) \]
The Neutral Kaons

Plotting this:

\[ P(\bar{K}^0) = \frac{1}{4} \left( e^{-\lambda_1 t} + e^{-\lambda_2 t} - 2e^{-(\lambda_1+\lambda_2)t/2} \cos(\Delta mc^2 t / \hbar) \right) \]

Note: This plot contains arbitrary values of \( \lambda_1 \), \( \lambda_2 \), and \( \Delta m \) to show the form of the result.
The Neutral Kaons

We see that if $\Delta m \neq 0$ there is an oscillation in the probability of seeing a $\bar{K}^0$ as a function of time.

The period of this oscillation is a function of the mass difference between the $K_1$ and $K_2$.

Measurements have shown that there is a mass difference.

Best Measurement is $\Delta m = 3.183 \pm 0.006 \times 10^{-6}$ eV/c$^2$

This is very small compared to the mass of a Kaon ($\sim 500$ MeV/c$^2$).

These oscillations are call hypercharge oscillations or strangeness oscillations, since it is the strangeness of the kaon that is oscillating.
The Neutral Kaons

The kaons provide a testing ground for CP invariance. By using a long enough drift distance for an initial beam of $K^0$ particles (i.e. long enough drift time), an arbitrarily pure beam of the long-lived $K_2$ kaons can be produced. Then if any decays to $2\pi$ are observed (which can only come from a $K_1$ decay) this is an indication of CP violation.

This was observed in 1964. It is a small effect but the result indicates that the long lived kaons are not purely $K_2$ but must have a small admixture of $K_1$. i.e. The K-long is

$$K_L = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_2\rangle + \varepsilon |K_1\rangle)$$

with $\varepsilon = 2.24 \times 10^{-3}$

Since then, additional examples of CP violation have been found. CP violation provides evidence of the unequal treatment of matter and antimatter and may be the reason why matter is dominant over antimatter in the universe.