Radiative captures
in
astrophysics and nucleosynthesis

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LOWq workshop

Low-E (or q!) captures are ubiquitous in nucleosynthesis:

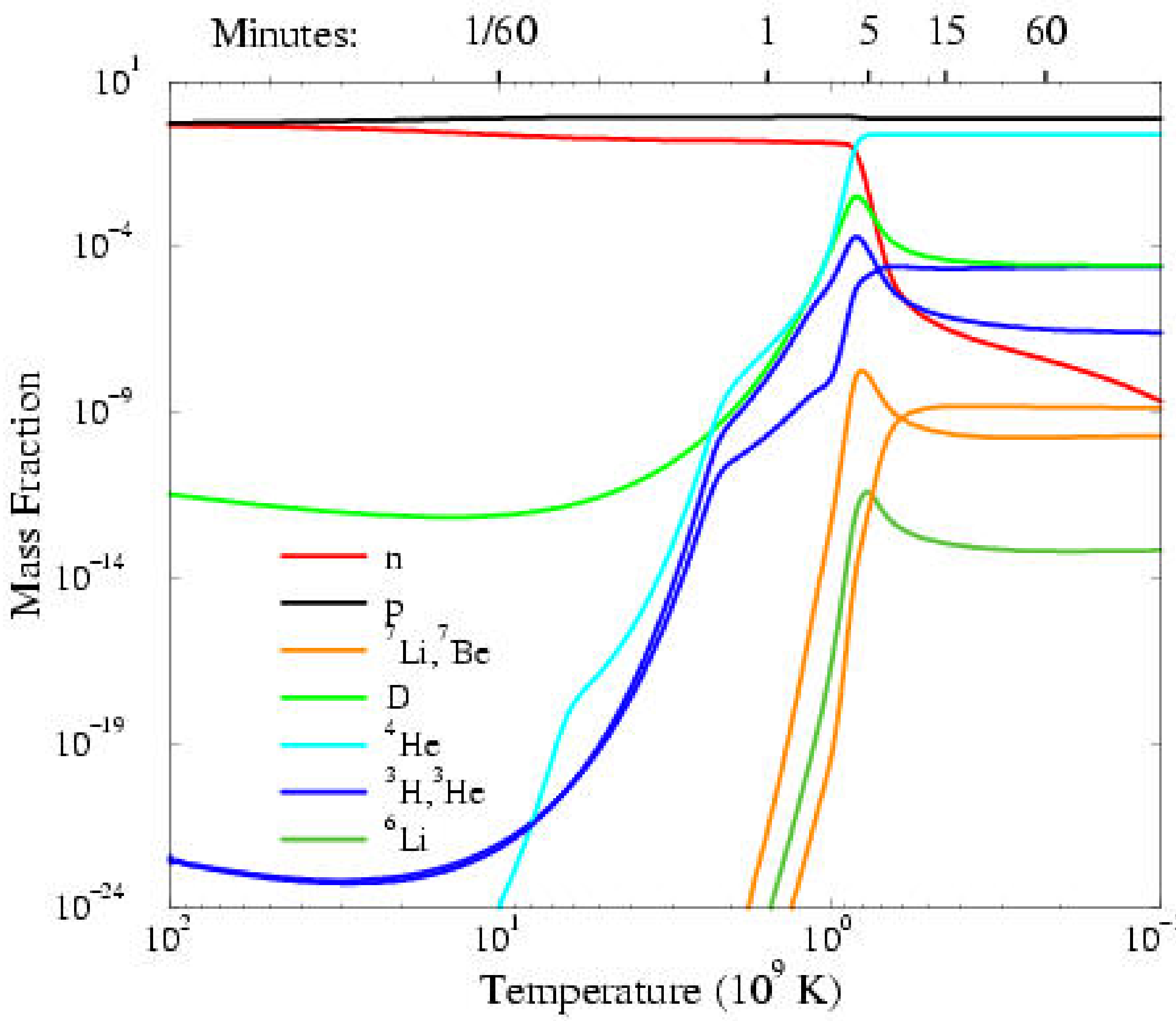
$p(n,\gamma)d$	starts nucleosynthesis
$^3H(\alpha,\gamma)^7Li$	bridge A=5 gap in BBN
$^3He(\alpha,\gamma)^7Be$	solar ν's
triple-α	bridge A=8 gap
$^{12}C(\alpha,\gamma)^{16}O$	sets stage for post-MS evolution
(p,γ)	CNO energy generation late stellar evolution novae, p-process
(n,γ)	all synthesis beyond Fe

I've been active in AGB evolution & primordial nucleosynthesis

Primordial lithium, Quantum Monte Carlo

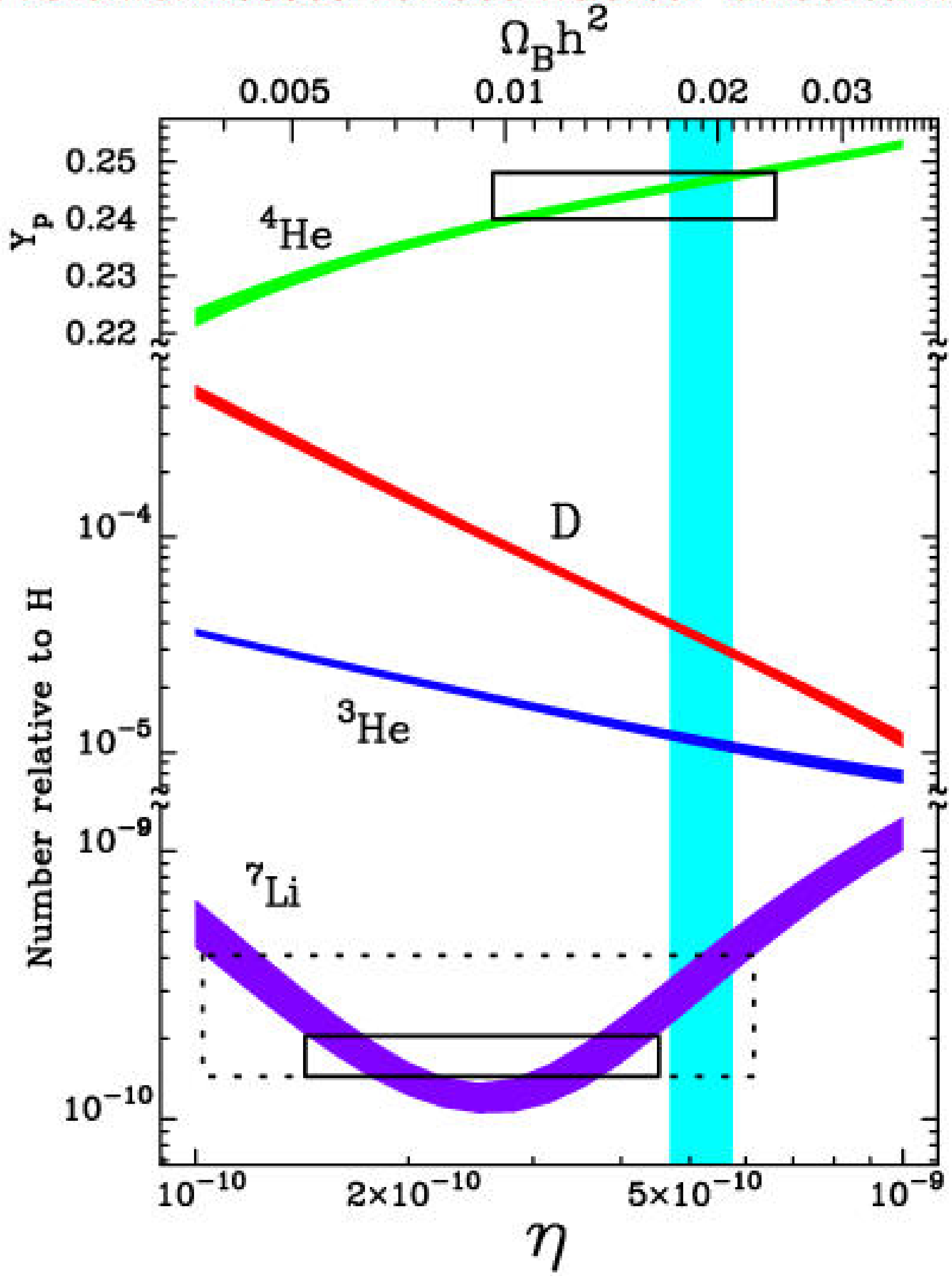
$\sim$ 1 second after beginning, $p \leftrightarrow n$ weak rates freeze out so $n_n/n_p \sim 1/6$ (Hayashi)
Then nuclear statistical equilibrium holds to $\sim$ 5 minutes

High entropy, few neutrons $\rightarrow$ only light nuclei
Almost all neutrons end up in ^{4}He
A few scraps left over: D, ^{3}He, ^{7}Li
at 10^{-10} to 10^{-5} rel. to H



Resulting abundances depend on mean baryon density

Curve thicknesses reflect nuclear uncertainties



Observed abundances tell about baryon density, Galactic evolution (and/or missing physics?)

Few-percent predictions for ^{7}Li/H potentially useful for sorting out BBN / chemical evolution / stellar models

Significant ^{6}Li not expected from BBN, but important to know for sure because it subsequently evolves along with ^{7}Li

Therefore crucial to have good rates for $d(\alpha,\gamma)^6$Li, ^{3}H$(\alpha,\gamma)^7$Li, ^{3}He$(\alpha,\gamma)^7$Be

Good data exist, and theory OK, but more information welcome

A good case to apply realistic potentials, since normalizations hard to measure and generally hard to predict *ab initio*

We (Nollett, Wiringa, Schiavilla 2001; Nollett 2001) set out to compute rates from Argonne v_{18}/ Urbana IX via Quantum Monte Carlo

Used Variational Monte Carlo (VMC) for wave functions, though Green's-function Monte Carlo better in principle

VMC easier to apply for 1st shot

Bound states derived by VMC:

^{3}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li, ^{7}Li*, ^{7}Be, ^{7}Be*

Deuteron is a direct solution

W.f.s based on previous work by Wiringa & collaborators

Reactions all peripheral at low $E \rightarrow$ need to ensure correct asymptotic form for p-shell nuclei (QMC insensitive to tails)

Previous work has shell-model-like w.f for p shell,

$$
\begin{aligned}
|\Psi_J\rangle \;=\; \mathcal{A}\Bigg\{ & \prod_{i<j<k\le 4} f^{sss}_{ijk} \prod_{n\le 4} f^{spp}_{n56} \prod_{i<j\le 4} f_{ss}(r_{ij}) \\
& \times \prod_{k\le 4} f_{sp}(r_{k5}) f_{sp}(r_{k6}) f_{pp}(r_{56}) \\
& \times \sum_{LS} (\beta_{LS} |\Phi_6(LSJMTT_3)_{1234:56}\rangle) \Bigg\}
\end{aligned}
$$

and

$$
\begin{aligned}
|\Phi_6(LSJMTT_3)_{1234:56}\rangle = & \\
& |\Phi_\alpha(0000)_{1234} \phi_p^{LS}(R_{\alpha 5}) \phi_p^{LS}(R_{\alpha 6}) \\
& \times \Big\{ [Y_{1m_l}(\Omega_{\alpha 5}) Y_{1m_l'}(\Omega_{\alpha 6})]_{LM_L} \\
& \times [\chi_5(\tfrac{1}{2}m_s)\chi_6(\tfrac{1}{2}m_s')]_{SM_S} \Big\}_{JM} \\
& \times [\nu_5(\tfrac{1}{2}t_3)\nu_6(\tfrac{1}{2}t_3')]_{TT_3}\rangle \;.
\end{aligned}
$$

p-shell single-particle functions were constructed with

$$r\phi_p^{LS}(r \to \infty) \propto j_1(kr), k = \sqrt{2\mu E}/\hbar,$$

correct where potential falls off faster than $1/r$.

Because there is clusterization (low separation energy, large spectroscopic factor), we enforce cluster behavior with

$$\left[\phi_p^{LS}(r)/r\right]^{n_p} \propto W_{km}(2\kappa r)/r$$

at large r.

Correlations between p nucleons chosen to produce deuteron/triton when far from s-shell core.

Binding energy improved 0.2 MeV for ^{6}Li, not changed for $A = 7$. Underbinding is a clear consequence of trouble with extremely compact configurations.

Overlaps with cluster states now have the expected form.

Continuum states written as (e.g.)

$$|\psi_{\alpha d}\rangle = \mathcal{A}\left\{\phi_{\alpha d}(r_{\alpha d}) Y_{LM_L}(\hat{\mathbf{r}}_{\alpha d}) \prod_{ij} G_{ij} |\psi_\alpha \psi_d^{m_d}\rangle\right\}_{LSJM},$$

ψ_α and ψ_d ground states

G_{ij} cluster distortions $\longrightarrow$ 1 beyond 1 fm, unimportant

$\phi_{\alpha d}(r_{\alpha d})$ resembles scattering w.f. between initial nuclei

Constructed as such, from potentials fitted to αd (or α ^{3}He) scattering data

Important qualities:

nodes (forbidden states) and P dependence for Pauli repulsion

location and width of αd d-wave states ($\mathbf{L} \cdot \mathbf{S}$ term)

$\longrightarrow$ deep Woods-Saxon plus Coulomb potentials

The Weakest Link – it would be better to derive from NN potential

Current operators

Current operators computed in standard LWA ^{3}He$(\alpha,\gamma)^7$Be and ^{3}H$(\alpha,\gamma)^7$Li essentially all the usual Siegert E1 operator, $E_1 \propto \sum_i Z_i \mathbf{r}_i$.

$L=0 \longrightarrow L=1$ or $L=2 \longrightarrow L=1$

$d(\alpha,\gamma)^6$Li is trickier

ground state $L=0$

$L=0 \longrightarrow L=0$ could go via $M1$

BUT αd relative coordinate does not appear in LWA $M1$, so matrix element is overlap of 2 orthogonal functions

$L=1 \longrightarrow L=0$ could go via $E1$

BUT operator looks like

$$E_1 \propto \left(\frac{Z_\alpha}{m_\alpha} - \frac{Z_d}{m_d}\right)\mathbf{r}_{\alpha d} \sim \left(\frac{2}{4} - \frac{1}{2}\right)\mathbf{r}_{\alpha d}$$

$L=2 \longrightarrow L=0$ $E2$ wins at most energies

Small corrections to LWA will be important at low E

Several corrections to LWA $E1$ operator could be important at astrophysical energies.
Moreover, data at 1-2 MeV show unexpected $E1$ strength

We considered: Relativistic correction to $\mathcal{O}(q)$ term
$\mathcal{O}(q^2)$ and $\mathcal{O}(q^3)$ terms in LWA
Pion charge density
Relativistic translation invariance

Largest piece:
Translation invariance enforced by working in CM coordinates
But we really want center-of-momentum coordinates, which differ by 10^{-2} fm – approximate by "center of energy"
$\longrightarrow$ include potential & kinetic energy in particle masses to compute CM (cluster models do this tacitly)

Only $\pi^{\pm}$ term completely negligible

Can't estimate most effects straightforwardly in a cluster model

Matrix element integration

VMC method:
Expectation values $\frac{\langle\psi_V|H|\psi_V\rangle}{\langle\psi_V|\psi_V\rangle}$ computed from weighted (Metropolis) Monte Carlo with weight function $W(\mathbf{R}) = (\psi_P|\psi_P)$ (no spatial integration) to match integrand approximately. $|\psi_P\rangle$ is an easier-to-compute version of $|\psi_V\rangle$.

Initially tried same procedure for DC matrix elements, weighting according to final state.

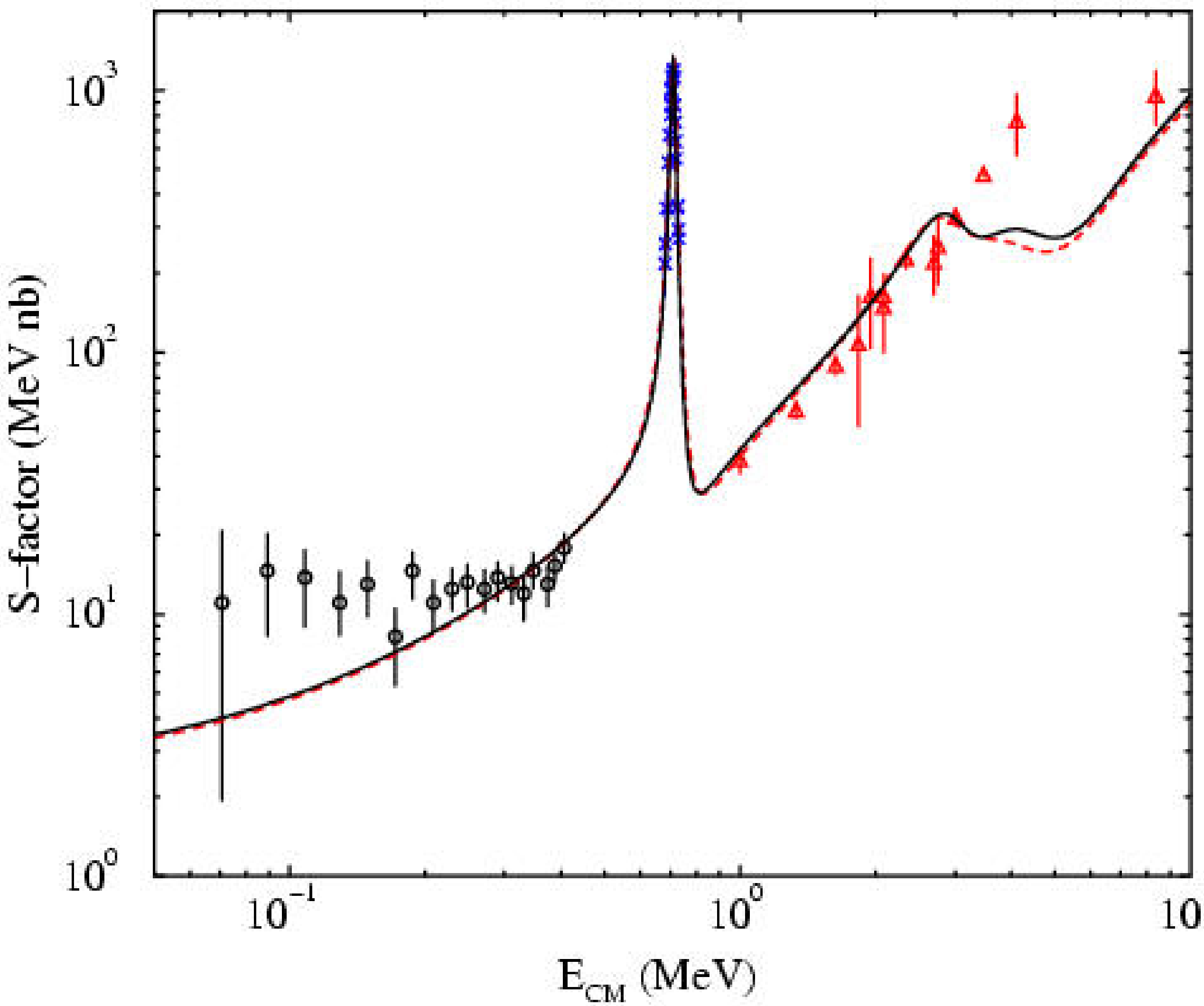
Proved impractical because this W falls off with cluster separation as $e^{-2\gamma r}/r^2$, while

$$\left(\psi_{^6\mathrm{Li}}|\mathcal{O}|\psi_{\alpha d}\right) \sim r^\lambda e^{-\gamma r}/r^2.$$

Tails sampled better by e.g. $W = \sqrt{\left(\psi_{^6Li}|\psi_{^6Li}\right)}$.

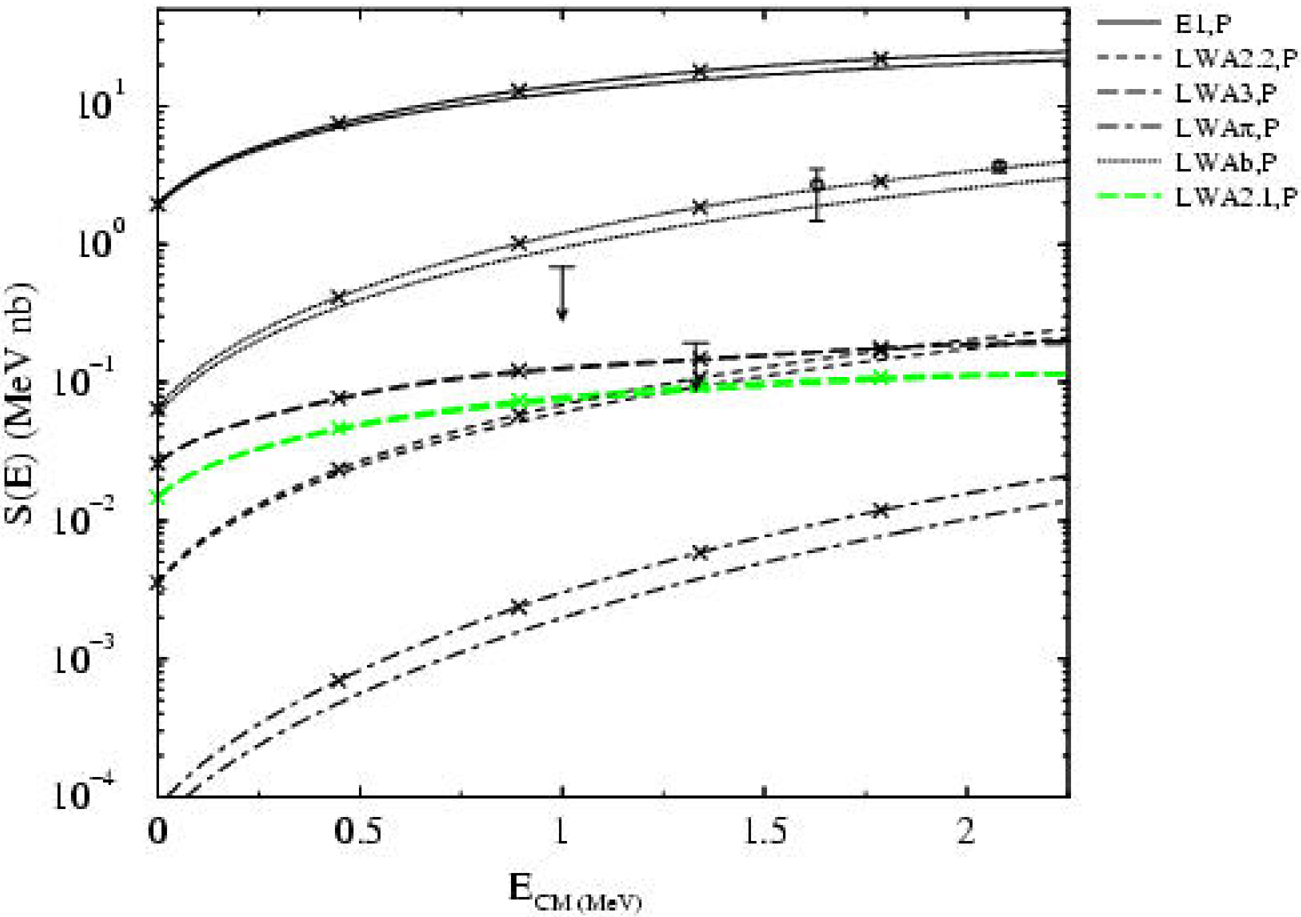
Excellent guess for $A = 6$, but stuck with $\sim$ 10% Monte Carlo errors on $A = 7$ results.

$d(\alpha,\gamma)^6$Li Results



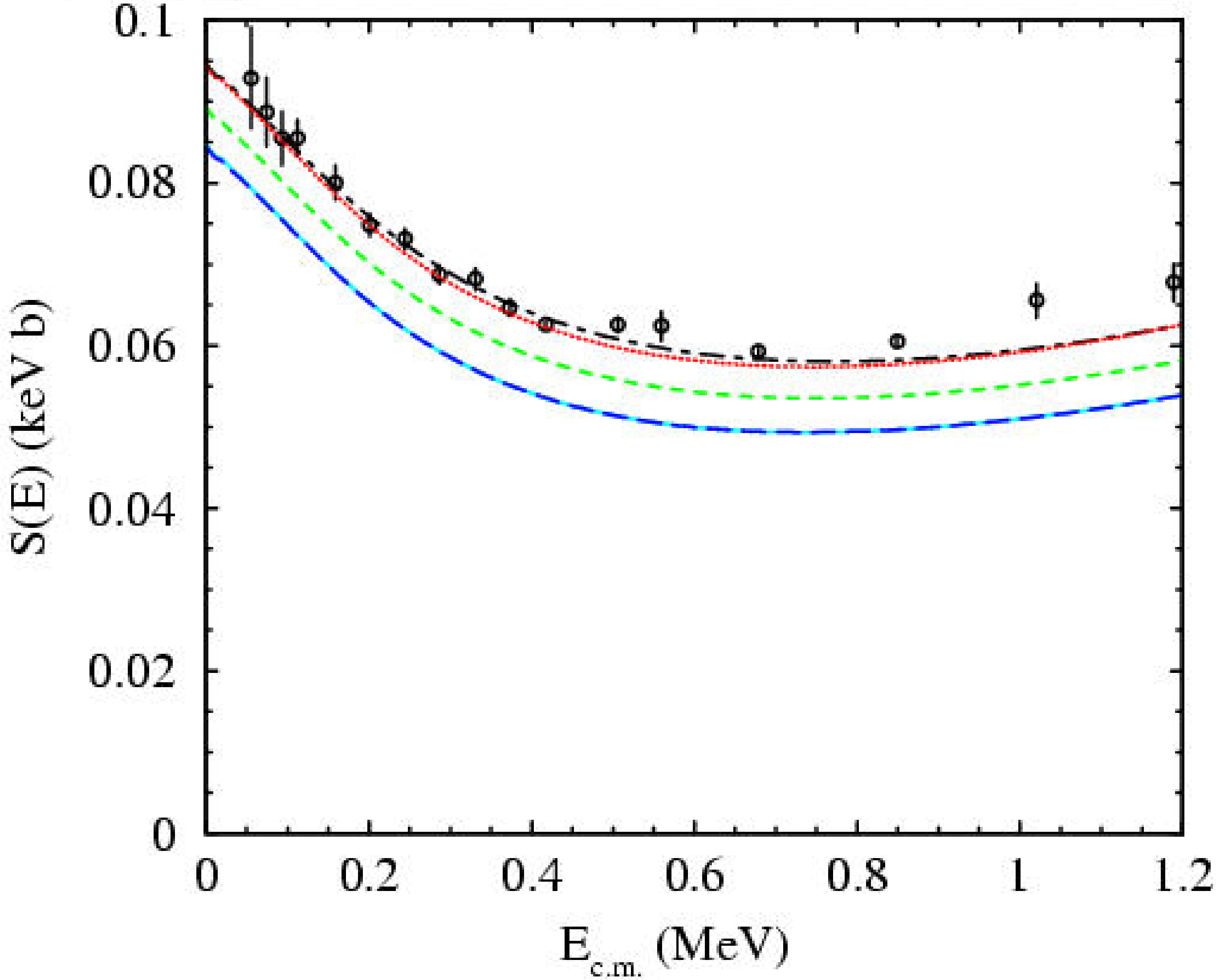
Looks much like other modern calculations

$d(\alpha,\gamma)^6$Li Results



Most of these corrections are interesting at low E, but puzzle of asymmetry data remains

$^3H(\alpha,\gamma)^7Li$ Results

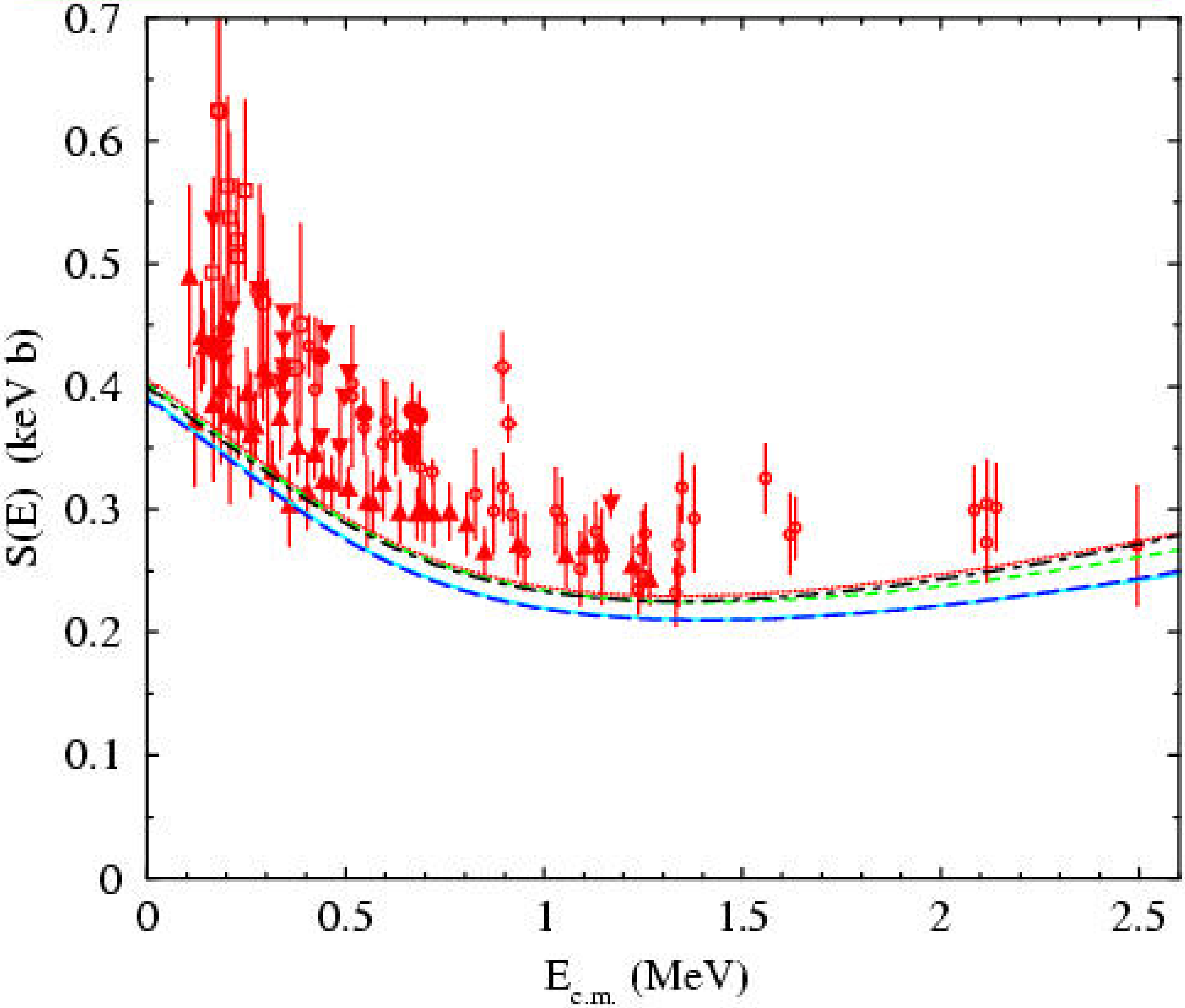


Data from Brune, Kavanagh, Rolfs (1993) (6% normalization uncertainty, not shown)

$\chi^2/\nu = 2.4$ after adjusting normalization

10% variation with choice of phenomenological continuum correlation, plus 10% Monte Carlo uncertainty

$^3He(\alpha,\gamma)^7Be$ Results



Energy dependence agrees well with data, previous models, no surprises

Branching ratio also agrees reasonably with data

$S(E)$ normalization more than 10% below any data set – factor of 2 below some – weak dependence on continuum choice

Lessons learned

QMC can be applied to $A = 6, 7$ α captures

Asymptotic forms improved, but more pressure to fix compact-configuration problem

QMC is useful tool for small corrections

Nothing obviously wrong with earlier cross section work – no indication of trouble for nucleosynthesis

Future prospects

Real continuum states

GFMC wave functions

Illinois NNN potentials

^{7}Be$(p, \gamma)^8$B, but only if we find a clever MC weighting strategy